# Sequential Optimization of an Emergency Response Vehicle's Intra-link Movement in a Partially Connected Vehicle Environment 

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## AUTHOR CONTRIBUTION STATEMENT

The authors confirm contribution to the paper as follows: Gaby Joe Hannoun: Conceptualization, Methodology, Programming, Writing - Original draft, Writing - Review and Editing, Visualization, Data curation, Investigation. Pamela Murray-Tuite: Conceptualization, Methodology, Writing - Review and Editing, Supervision, Project administration, Funding acquisition. Kevin Heaslip: Conceptualization, Writing - Review and Editing, Supervision, Project administration, Funding acquisition, Data curation.

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#### Abstract

This paper introduces a semi-automated system that facilitates Emergency Response Vehicle (ERV) movement through a transportation link by providing instructions to downstream non-ERVs. The proposed system adapts to information from non-ERVs that are nearby and downstream of the ERV. As the ERV passes stopped non-ERVs, new non-ERVs are considered. The proposed system sequentially executes integer linear programs (ILPs) on transportation link segments with information transferred between optimizations to ensure ERV movement continuity. This paper extends a previously developed mathematical program that was limited to a single short segment. The new approach limits runtime overhead without sacrificing effectiveness and is more suitable to dynamic systems. It also accommodates partial market penetration of connected vehicles using a heuristic reservation approach, making the proposed system beneficial in the short-term future. The proposed system can also assign the ERV to a specific lateral position at the end of the link, a useful capability when next entering an intersection. Experiments were conducted to develop recommendations to reduce computation times without compromising efficiency. When compared to the current practice of moving to the nearest edge, the system reduces ERV travel time an average of 3.26 seconds per 0.1 mile and decreases vehicle interactions.


Keywords: Emergency services; optimization; intelligent transportation systems; connected vehicles.

## INTRODUCTION

Emergency Response Vehicles (ERVs) face numerous challenges when navigating to and from emergencies. ERV preemption adjusts the signal timings so that approaching ERVs do not wait in a queue at red signals and do not collide with vehicles entering the intersections from the opposing approaches. Conversely, the challenges experienced when travelling along transportation links need more investigation as the current practice is still limited to visual and audible warning systems (i.e., lights and sirens) that often fail to prevent confusion and vehicle collisions (De Lorenzo and Eilers, 1), especially for police vehicles (Missikpode et al., 2). Downstream vehicles should coordinate their movements to cooperatively open a path for the ERV to reach its destination as quickly and safely as possible.

Hannoun et al. (3) presented a mathematical program to facilitate ERV passage along a transportation link of predefined length by relying on vehicular communications. The integer linear program (ILP) identifies the fastest ERV intra-link path along with the ERV's maneuvering instructions based upon downstream information. The ILP also assigns to each downstream non-ERV a specific stopping position along the link. In Hannoun et al. (3), the proposed system is limited to short links due to computation time, and so is only effective if activated by the ERV driver needing assistance over a short distance. However, the ERV's driver may request extended assistance, especially in congested urban transportation networks. To practically facilitate the passage of the ERV along its complete path from origin to destination, an updated approach requires solutions to the following challenges left unaddressed from the previous methodology: (1) substantial computation time due to the considerably large problem size and (2) unnecessary early notification and instruction dissemination to non-ERVs travelling downstream on the ERV path and distant from the ERV position. Furthermore, this study relies on connected vehicle technologies for vehicular data collection and dissemination of the instructions to ERVs and non-ERVs. Although the connected vehicle environment is rapidly evolving, the deployment of its hardware and software components remains challenging. The market penetration rate of connected vehicles will remain low in the near-term future (Feng et al., 4) which prevents the work in Hannoun et al. (3), in which all vehicles are connected, from being directly applicable in the near future.

The contributions of this paper are threefold. First, the problem size issue is addressed and the facilitation of the ERV's movement along larger links is made possible with the introduction of a sequential approach along with recommendations about how to apply it to minimize computation time without compromising the system's efficiency. Second, this paper adds more flexibility into the optimization model by allowing the assignment of the ERV to a desired specific lateral position along the link, a useful capability when next entering an intersection or navigating to a road accident. Third, the assumption of full market penetration is resolved by introducing a technique that estimates the presence of unconnected vehicles and reserves additional space for the unconnected vehicles.

The remainder of this paper is divided into six sections. The first section briefly reviews previous studies proposing systems for ERV support and techniques mitigating partial market penetration in connected vehicle applications. The second section presents the system proposed in this paper and the third section outlines the preprocessing steps required prior to each optimization. In the fourth section, the adjusted ILP formulation is introduced while the experimental plan and results from the evaluation of the system with partial mar-
ket penetration and from comparison with current practices are included in the fifth section. Finally, the sixth section reviews the main points of the paper and suggests applications and extensions for future work.

## LITERATURE REVIEW

Connected vehicles allow the development of new applications focusing on enhancing safety, mobility, and/or the environment by relying on the exchange of real-time information of equipped vehicles with each other, roadside infrastructure, and the Internet. The connected vehicle environment should be leveraged to improve ERVs' operations on the roads as "crashes involving emergency vehicles . . . are a substantial problem nationwide" (Savolainen et al., 5). Assistance can be provided to emergency responders in several forms such as using automatic crash notification (Fogue et al., Fogue et al., Martinez et al., 6, 7, 8) and intersection management systems (Dresner and Stone, Cetin and Jordan, 9, 10). Yet, it is important to develop smart solutions that mitigate the challenges associated with the ERVs' movement along transportation links as confusion is a serious issue that downstream vehicles face upon hearing the siren and/or visually detecting an ERV. Emergency vehicle warning systems support emergency services as they alert downstream and surrounding vehicles of the presence of a nearby ERV, granting these vehicles more time and relevant information to react efficiently (Buchenscheit et al., 11). However, these alert systems neither suggest nor recommend the best actions to be adopted by downstream vehicles. Non-ERVs may still fail to respond in a timely manner and to coordinate their movements with the other vehicles adequately, resulting in a slower ERV movement. In many cases, downstream traffic should change lanes to provide a free passage for the ERV. Studies investigating the impact of lane changes on capacity (Chen and Ahn, 12) and proposing lane change advisory systems (Ramezani and Ye, Tilg et al., Wang et al., 13, 14, 15) using automated vehicle and/or connected vehicle technologies exist. The optimization of the ERV movement can be regarded as a special type of lane change advisory problem that gives priority to ERVs. A road reservation system on a two-lane link facilitates the passage of an ERV by requesting the downstream vehicles to shift away from the lower density lane, reserving it for ERV use (Yoo et al., 16). The limited road width and assumption that the lower density lane is the best intra-link ERV path constitute limitations as the ERV may desire the use of a specific lane based on its next intersection movement and/or the emergency scene's location. Another system inducing a lane change maneuver for vehicles obstructing the ERV's way is proposed by Weinert and Düring (17). While a rescue lane for the ERV is freed and improvements in the ERV travel time are observed, this system assumes a predefined ERV path (i.e., it does not generate the best ERV intra-link). Furthermore, the ERV does not receive recommendations about the best maneuvering actions based on the feasible cooperative movements that downstream vehicles can acquire. Similarly, in Toy et al. (18), priority is granted to ERVs on an automated highway system by moving vehicles or platoons of vehicles out of the ERV's way. The ERV only reacts to the available downstream space that can be provided by the downstream vehicles and its movement may not be optimal. In addition, the study does not accommodate the presence of unconnected (unequipped) vehicles.

The main capabilities of connected vehicles are data availability and exchange. To evaluate the performance of connected vehicle applications, mitigating partial market penetration is crucial. A partial market penetration means omitted information as the unequipped
vehicles are not sharing their corresponding data. The basic idea relies on estimating the positions of the unequipped vehicles using data received from equipped vehicles (Feng et al., 4). A microscopic estimation is introduced in (Goodall et al., 19), where the state of unconnected vehicles is obtained after comparing the actual and expected behaviors of the connected vehicles pairs. Feng et al. (4) define the queuing, slow-down and free-flow regions along the link upstream of the intersection and use a different algorithm to estimate the unconnected vehicle presence and status in each. In the slow-down region, a rule inspired by (Park, 20) is adopted in which an unconnected vehicle is inserted when the observed headway between two consecutive equipped vehicles is unusual and exceeds a given threshold based on the Wiedemann car following model. The estimation technique used in this paper is built on this rule. Finally, it is important to note that the strategies used to account for partial market penetration can also be employed to consider imperfect human compliance and degraded vehicle communications. For instance, an indirect decrease in the market penetrations may result as equipped vehicles lose connectivity (Feng et al., 4).

## PROPOSED SYSTEM

In this paper, the initial approach of Hannoun et al. (3) is extended to optimize the ERV passage over larger distances. The approach assumes the presence of a centralized computing server that preprocesses the collected data (positions, speeds, and deceleration capabilities of connected vehicles), runs the ILP, post-processes and stores the ILP output. This server is assumed to send messages to the non-ERVs at a time that ensures their smooth arrival to their final respective positions before the ERV's passage. As such, a warning module that determines when to notify downstream non-ERVs is assumed to be embedded in the server. The proposed ILP formulation, in its present form, only accommodates a single ERV and is limited to a transportation link with no intersections or non-ERVs entering/leaving the link.

## System Description

The roadway, including traversable shoulders, is discretized into identical cells of size L by W , each of which is the size of a regular vehicle plus a buffer. A cell is characterized by its $(x, y)$ coordinates, where the X -axis denotes the longitudinal motion (i.e., direction of flow) and the Y-axis refers to the lateral motion (i.e., lane changes). The ERV instructions are generated at every increment $i$, where one increment encompasses a number of cells equal to the ERV longitudinal size plus a buffer of 1 . The ERV speeds associated with each increment are expressed in speed stage to take integer values while avoiding impractical acceleration and deceleration rates (see the Appendix in (Hannoun et al., 3)). The non-ERVs are labeled by $j$ based on their initial positions; non-ERVs located on higher $x$ and $y$ coordinates receive larger labels.

## System Model

As shown in Figure 1, the link is divided into segments called Initial Ranges (IRs). An ILP is executed for each IR after preprocessing the collected data from the non-ERVs present in this IR. Each ILP generates the ERV optimal intra-link path and non-ERV positions along a downstream range called the Assignment Range (AR). The location of the AR along the link is dictated by the minimum stopping distance of the non-ERVs travelling on its corresponding

IR, thus overlap and/or gaps may appear among consecutive ARs. Subsequently, the system performs a series of preprocessing steps to ensure continuity from one optimization to the other (discussed further in the Preprocessing Steps section).


FIGURE 1: Link segment divided into IRs

## Partial Market Penetration

Each connected non-ERV receives an instruction message with a location at which it should stop. An assignment message is vehicle specific which means that variable message signs (VMS) cannot be used to assist unconnected non-ERVs. In this study, it is assumed that the unconnected non-ERV drivers are alert and aware of the approaching ERV (use of sirens) and that each of these non-ERVs follows the behavior of the non-ERV in front of it and in the same lane regardless of whether it is equipped or unequipped. This assumption resembles the one used in (Jiang et al., 21) that consists of adopting a car following model as a control system to monitor the unconnected vehicles. As unconnected non-ERVs are also unable to share their information, a technique is developed for the estimation of their positions based on the available information received from the connected vehicles. The ILP is independent of the estimation technique, so other advanced approaches, such as those relying on sensor data, can be used instead of the following approach:

1. Determine all the possible positions of unconnected non-ERVs according to a distance-based criterion adopted for the slow-down region in (Feng et al., 4). This step identifies the maximum number of unconnected vehicles that can be added between connected non-ERVs. Hence, inputting all these unconnected non-ERV positions would result in higher traffic flow than the actual one and should be avoided.
2. Estimate the maximum number of unconnected non-ERVs $(\chi)$ that should be present on the link knowing that the total (unconnected and connected) number of non-ERVs is equal to the number of connected (observed) non-ERVs divided by the market penetration level.
3. Randomly select $\chi$ positions from the unconnected non-ERV positions determined in Step 1.

In the long-term future, a connected vehicle is expected to have an autonomous vehicle's capability of sensing nearby non-ERVs, hence making the estimation of unconnected nonERVs more accurate.

## PREPROCESSING STEPS

Prior to each ILP optimization, a set of preprocessing steps are required to prepare the corresponding input data. Some of these steps are solely based on characteristics of the IR undergoing optimization next, while other preprocessing steps are basically retrieving output from the last optimization and converting it into input for the next one. Parameter and variable notation are listed in Table 1.

## Based on the IR's characteristics

These preprocessing steps consist of defining, for each IR, (1) the feasible stopping range (FSR) of each non-ERV travelling on it, (2) its corresponding AR's starting/ending positions and (3) the presence of unconnected vehicles along with their corresponding leader. The FSR of each non-ERV is defined to make sure that each non-ERV is assigned to a location it can comfortably reach. It is based on the minimum stopping distance which accounts for the distance travelled by the non-ERV during the time for communication and computation which is the time elapsed between the data collection and receipt of the instruction. The FSR of each non-ERV extends $c$ cells beyond its minimum final position to limit the optimization space. (The FSR longitudinal size or FSR cut-off value $c$ is an input parameter to the ILP). The optimization takes place on the AR which is downstream of its corresponding IR and includes the FSRs of all non-ERVs in this IR. To obtain a discrete number of ERV instructions in the AR, the longitudinal size of the AR is increased, only if needed, so that it is always a multiple of an increment size (i.e. ERV longitudinal size + buffer). The previously discussed estimation technique should be implemented prior to each IR optimization when less than full market penetration exists. The ILP in (Hannoun et al., 3), left unchanged, will fail to consider that the unconnected non-ERVs cannot receive messages with enclosed positions. In this paper, the ILP is adjusted such that a following behavior between a predicted unconnected non-ERV and its preceding non-ERV is imposed to monitor the action of the former. So, along with the estimated positions of unconnected non-ERVs, the leader ( $l e$ ) of each is identified and input in the ILP.

## Based on previous optimization's output

These steps are crucial in building continuity from one optimization to the other. First, the length of the FSR (c) should be determined. As previously discussed, each non-ERV is allowed to stop within its corresponding FSR which starts at its MFP and which extends $c$ cells beyond the MFP. The higher the $c$ value, the larger the problem size, as the total AR's longitudinal size (optimization space) becomes higher by default. In cases when the ILP fails to generate an optimal solution using a given $c$ value, re-running the ILP with a higher $c$ value may help find a solution as non-ERVs have more final position alternatives and have more space along which to spread. The proposed system increases the $c$ value incrementally until a solution is found, yet, these iterations may be computationally intensive. Hence, a new preprocessing step is added to skip the unnecessary iterations and reach optimality faster. It dynamically (i.e., depending on scenario-specific parameters) finds a minimum initial $c$ value using simple computations based on rules of thumb. For example, a minimum $c$ value is found such that (1) the available number of cells for the ERV movement is at least equal to the one required (2) the FSR of a given vehicle does not start after the end of the FSR of another downstream vehicle as passing is prohibited
(Appendix available upon request). Second, the ERV initial lateral position and speed have to be deduced from the previous optimization's output. If the current AR overlaps with the previous AR, the ERV initial lateral position and speed generated at the increment in the previous AR that coincides with the first increment of the current AR is retrieved. If a gap exists between the current and previous ARs, the ERV initial lateral position and speed are deduced by assuming that the ERV maintains a straight path after exiting the previous AR and increases its speed linearly up to the maximum allowable ERV speed ( $\left.S^{\text {free }}\right)$. Third, to ensure that passing is prohibited among non-ERVs in different ARs, a binary parameter $\left(n p^{x, y}\right)$ at each cell $(x, y)$ in the current AR is determined. It takes the value of 1 at and before the position of the most downstream vehicles in the previous AR and 0 otherwise. Fourth, to avoid confusion in case of overlap, previously generated ERV instructions delineating the ERV path up to the most downstream non-ERV position in the previous AR are maintained. A binary parameter $\left(\partial_{k}^{i, y}\right)$ is set to 1 if an ERV instruction $k$ should be maintained at increment $i$ and lateral index $y$ and 0 otherwise. Fifth, and as previously discussed, when the current AR overlaps with the previous one, the ERV initial speed in the current AR is retrieved from the previous output. Yet, the ERV speed at each increment in this AR should also take into consideration the presence of non-ERVs along the overlap. The $\left(s u^{i}\right)$ parameter at increment $i$ along the overlap takes the value of the speed that is obtained at the same location in the previous AR and only limited by the number of surrounding non-ERVs (called the speed environment). When the overlap ends or in cases when overlap does not exist, this parameter takes a large value (i.e. infinity).

The execution of these preprocessing steps should preferably start with the determination of the minimum length of the FSR (i.e., first step in the second group), then should be followed by the identification of the FSR of each non-ERV and the AR (i.e., first and second steps in the first group). The remaining steps can be performed without following any particular order.

## MATHEMATICAL FORMULATION

The objective function (Equation 1) of each IR optimization maximizes the ERV's speeds $\left(s^{i}\right)$ at each increment (first component in Equation 1), maximizes the number of free cells adjacent to the ERV path through the maximization of ( $s_{\text {env }}^{i}$ ) at each increment (second component in Equation 1), and minimizes the longitudinal indices of the final nonERV positions (third component in Equation 1). There may be alternate optima for each scenario. Alternative optimal solutions may have different non-ERV positions that lead to the same ERV path and ERV speeds. Since passing among non-ERVs is prohibited, the alternative optimal solutions with the more upstream final non-ERV positions is preferred and selected. This way the non-ERVs of the following IR optimization are allowed to stop on cells with smaller longitudinal indices, hence more efficiently utilizing the downstream space. The summation of $\left(v_{j}^{x, y} x\right)$ is multiplied by a very small factor $\left(\alpha_{3}\right)$, so that the third component favors one alternative solution (the one with the smallest sum of non-ERV longitudinal positions) without having an impact on the selection of the optimal ERV path and ERV speeds. Equal weights are assigned for the first and second components $\left(\alpha_{1}\right)$ and $\left(\alpha_{2}\right)$ respectively as this is the most unbiased combination based on the weight analysis performed in (Hannoun et al., 3). The ERV speeds generated by the system and maximized

TABLE 1 : Parameter and Variable notation

| Notation | Description |
| :---: | :---: |
| $N$ | ERV longitudinal size (in cells) |
| LL | Longitudinal size of the AR (in cells) |
| Y | Lateral size of the AR, including traversable shoulders (in cells) |
| $J$ | Number of non-ERVs in the IR |
| $c$ | Longitudinal FSR cutoff value (in cells) |
| $n p^{x, y}$ | Binary parameter that takes the value of 1 at cells with a longitudinal index $x$ less than or equal to the longitudinal index of the most downstream non-ERV in the previous AR and 0 otherwise |
| $\partial_{k}^{i, y}$ | Binary parameter that takes the value of 1 when an ERV instruction $k$ should be at increment $i$ and lateral index $y$ imposed and 0 otherwise |
| $\ddot{Y}$ | Final desired ERV lateral position |
| $\xi$ | Binary parameter that takes the value of 0 when a final ERV lateral position is desired and 1 otherwise |
| $t y_{j}$ | Binary parameter that takes the value of 1 if non-ERV $j$ is a connected vehicle and 0 otherwise |
| $l e_{j}$ | Integer parameter referring to the label of leader of each non-ERV $j$ in the IR |
| $w^{x, y}$ | ERV assignment binary variable equal to 1 if cell $(x, y)$ is included in the ERV path and 0 otherwise (where $x=1, . ., L L$ and $y=1, . ., Y$ ) |
| $s^{i}$ | ERV speed integer variable referring to the ERV speed at increment $i$ (where $i=1, . ., L L /(N+1)$ ) |
| $d_{k}^{i, y}$ | ERV instruction binary variable equal to 1 when instruction $k$ is given to the ERV at increment $i$ and lateral position $y$ (where $i=1, . ., L L /(N+1)-1 ; y=1, . ., Y$; $k=1$ means move right; $k=2$ means go straight and $k=3$ means move left) |
| $v_{j}^{x, y}$ | Non-ERV assignment binary variable equal to 1 if cell $(x, y)$ is allocated to non-ERV $j$ cell and 0 otherwise (where $x=1, . ., L L ; y=1, . ., Y$ and $j=1, . ., J$ ) |
| $s_{e n v}^{i}$ | ERV speed environment integer variable referring to the ERV speed at every increment $i$ only based on the ERV surrounding (where $i=2, . ., L L /(N+1)$ ) |

in the objective function are not disseminated to the ERV. The ERV driver only receives the intra-link path (through maneuvering instructions) and increases/decreases or maintains its speed when required or desired.

Maximize $z=$

$$
\begin{equation*}
\alpha_{1} \sum_{i=2}^{L L /(N+1)} s^{i}+\alpha_{2} \sum_{i=2}^{L L /(N+1)} s_{e n v}^{i}-\alpha_{3} \sum_{x} \sum_{y} \sum_{j}\left(v_{j}^{x, y} x\right) \tag{1}
\end{equation*}
$$ previously introduced are briefly listed below (for more details refer to (Hannoun et al., 3)):

- Each cell can be occupied by only one vehicle.
- In the AR, each non-ERV is allocated to exactly one cell which is in its corresponding FSR.
- Passing and weaving among non-ERVs in the same AR is prohibited.
- A passing lane that consists of one empty cell at every $x$ is freed for the ERV.
- Only one ERV instruction is generated at each increment.
- No right/left lane change is allowed when the ERV is on the rightmost/leftmost lane.
- ERV assignment and ERV instruction variables are linked to ensure continuous longitudinal and lateral motion.
- The cells constituting the ERV path should not be occupied by non-ERVs. It is assumed that an ERV path with lane changes needs additional cells to comfortably maneuver from a lane towards the other.
- The ERV speed environment $\left(s_{\text {env }}^{i}\right)$ is constrained by the number of nearby stopped vehicles around its next movement. The ERV speed environment variables, that only take into consideration the surroundings of the ERV, is defined separately to be maximized in the objective function so that the vehicles are directed away from the ERV's intra-link path even when the ERV speed has to decrease due to other factors such as lane change.
- The ERV speed $\left(s^{i}\right)$ takes into account the ERV's surrounding such as $\left(s_{e n v}^{i}\right)$ as well as the ERV instruction given at the previous increment. It is assumed that the ERV decreases its speed after performing a lane change while it can increase its speed if the ERV goes straight. Also, $s^{i}$ is bounded to the minimum and maximum allowable ERV speeds.

The new constraints are described in detail below:

- Passing among vehicles in different ARs is prohibited by Equation 2. For instance, non-ERVs in the current optimization can only stop after the most downstream non-ERV position of the previous AR. The $n p^{x, y}$ is a parameter that takes the value of 1 when the longitudinal index $x$ of the cell $(x, y)$ is less than or equal to the most downstream non-ERV longitudinal position in the previous AR and 0 otherwise.

$$
\begin{equation*}
\sum_{j}^{J} v_{j}^{x, y}+n p^{x, y} \leq 1 ; \quad \forall(x, y) \tag{2}
\end{equation*}
$$

- In the case of overlapping ARs, the ERV instructions generated from the previous AR at the same increment $i$ and occurring before the most downstream non-ERV position in the previous AR, are maintained using Equation 3. Subsequently, in
case of AR overlap, the previously obtained ERV path up to the most downstream non-ERV in the previous AR is maintained and the remainder of the path is subject to change as the ERV navigates a link section where new non-ERVs from the current AR will stop (Appendix available upon request). The $\partial_{k}^{i, y}$ is a parameter that takes the value of 1 if instruction $k$ should be imposed at increment $i$ and lateral index $y$, and 0 otherwise.

$$
\begin{equation*}
d_{k}^{i, y} \geq \partial_{k}^{i, y} ; \quad \forall k ; \forall i=1, \ldots, L L /(N+1)-1 ; \forall y \tag{3}
\end{equation*}
$$

- In many cases, the ERV has to exit the AR from a specific lateral position. A new constraint (Equation 4) is added to impose the desired last ERV lateral position $(\ddot{Y})$ when needed. The parameter $\xi$ takes the value of 0 when a final ERV lateral position is desired and 1 otherwise.

$$
\begin{equation*}
w^{L L, \ddot{Y}}+\xi \geq 1 \tag{4}
\end{equation*}
$$

- The technique adopted to account for partial market penetration requires the addition of a new constraint that implies following behavior by unconnected nonERVs. Equation 5 ensures that if the leader of an unconnected non-ERV $j$ stops at cell $(x, y)$ (i.e., $t y_{j}=0$ and $v_{l e_{j}}^{x, y}=1$ ), then, the non-ERV $j$ has to stop in the same lane $y$ (i.e., $\sum_{x^{\prime}=1}^{x-1} v_{j}^{x^{\prime}, y}=1$ ) to avoid passing among vehicles, but not necessarily in the cell directly upstream. Equation 5 ensures that only unconnected non-ERVs follow their respective leader regardless of the leader's type. In the case of full market penetration, all non-ERVs have $t y_{j}=1$, hence this constraint is not binding.

$$
\begin{equation*}
\sum_{x^{\prime}=1}^{x-1} v_{j}^{x^{\prime}, y}+t y_{j} \geq v_{l e_{j}}^{x, y} ; \quad \forall j ; \forall x=2, \ldots, L L ; \forall y \tag{5}
\end{equation*}
$$

- The initial ERV lateral position and speed at the start of the AR are determined based on the preprocessing steps and corresponding constraints are included accordingly.


## EXPERIMENTAL ANALYSIS

In this section, the sequential optimization is examined to provide insights and recommendations that are useful when applying this approach. The focus lies in evaluating new considerations for the sequential optimization: (1) the number/size of IRs in a link and (2) the grouping of vehicles in IRs. The IR number and IR size are two inversely proportional parameters; as the size of the IR increases the number of IRs decreases for a predefined link length. Hence, both parameters are evaluated in a single test (Test A). Furthermore, the components that characterize an efficient grouping of non-ERVs in IRs are evaluated in Test B. In addition, a sensitivity analysis (Test C ) is conducted to evaluate the performance of the system when a portion of the downstream non-ERVs is unconnected. The performance of the model (in which the ILP is executed with the estimated unconnected non-ERVs positions) is investigated by assessing the implications engendered after replacing the set of
estimated unconnected non-ERVs by the set of actual unconnected vehicles. Market penetrations ranging from $70 \%$ to $100 \%$ are tested for increasing $\mathrm{v} / \mathrm{c}$ (volume to capacity) ratios of $0.75,0.85$ and 0.95 . In Test D , for each $\mathrm{v} / \mathrm{c}$ ratio, the output is compared to the local practice "go to the nearest edge" and benefits in terms of ERV travel times are computed. The sequential optimization is coded in java with the AMPL API. The ILP is solved using the CPLEX solver. The java code is executed on a MacOS machine with a 3.1 GHz Intel Core i5 processor and 8 GB 2133 MHz LPDDR3 memory.

## Test A: IR size/number

The approach in (Hannoun et al., 3) could not find solutions within a reasonable amount of time for longer segments, so, the approach presented in this paper divides the link into smaller link segments (called IRs) that undergo an ILP optimization sequentially. This test consists of varying the number of IRs within 1575 ft (equivalent to 75 longitudinal cells) for three $\mathrm{v} / \mathrm{c}$ ratios $(0.75,0.85$, and 0.95$)$. Although the homogeneity of IR size is not mandatory in the approach, the sizes of the IRs in this test are the same and increase/decrease uniformly. As the IR size decreases, the problem size of a single IR drops because the number of decision variables and constraints declines. According to Figure 2, as expected, improvements in computation times are obtained as the link is divided into smaller IRs. The higher the $\mathrm{v} / \mathrm{c}$ ratio, the more significant the average computation time's improvement. However, for all v/c ratios, switching from 10 IRs to 15 IRs did not lead to noticeable computation time decreases, meaning that further reduction of IR sizes is not worthwhile. Decreasing the number of IRs does not guarantee benefits as it may jeopardize the optimized ERV path. Determining the optimal ERV path on consecutive short link segments may not be the same as the global optimal ERV path on larger link segments. In the tested scenarios, the decrease in IR size did not generate different ERV paths/ERV speeds. Nevertheless, even if the decrease of IR sizes does not impact the ERV path, one should select the minimum IR size beyond which no improvements in computation times are observed. Smaller IR sizes mean a larger number of IRs/ILP optimizations (within a predefined link) and this, in return, translates into higher volumes of data exchange over the network that potentially lead to more communication failures and packet losses.

## Test B: Non-ERVs' grouping in IR

This test consists of varying the grouping of non-ERVs while maintaining the same number of IRs in the 1575 ft link. The goal of this test is to demonstrate the implications that different groupings of vehicles in IRs have on the computation times. Different grouping patterns were tested and results showed that including large gaps (of a size at least equal to half of the IR size) in the IRs added to the computation times but did not affect the optimized ERV path. Consequently, this test acts as a confirmation that to efficiently group non-ERVs in IRs, large gaps should act as delimiters between IRs, and thus should not be included in any optimization.

## Test C: Market penetration tests

According to (Hannoun et al., 3), when an ERV is initially on the rightmost lane, more benefits in terms of ERV speeds were obtained. Hence, the scenarios in the following tests have the ERV entering from the rightmost lane. Three levels of congestion (v/c ra-


FIGURE 2: Average computation time per IR for different numbers of IRs within the link and different levels of congestion
tios) are evaluated: $0.75,0.85$ and 0.95 . For each level of congestion, market penetration levels ranging from $70 \%$ to $100 \%$, (at $10 \%$ increments) are tested. Based on the positions of the connected non-ERVs, the presence of unconnected non-ERVs is estimated using a distance criterion, as previously discussed. Next, the implications that the presence of the actual, unconnected non-ERVs have on the generated ERV movement and the connected non-ERVs following their corresponding instructed message are investigated. In this paper, it is assumed that the unconnected vehicles start following the behavior of their leader by following their lane changes and decelerating upon hearing the siren. As for the connected non-ERVs, it is assumed that they are well-aware of their surroundings and react in a way to accommodate unconnected non-ERVs blocking the ERV's path. These vehicles intuitively and slightly adjust their instructed positions, when possible, after perceiving a non-ERV blocking the ERV's passage. (The consideration of a more aggressive behavior of unequipped non-ERVs is important in ongoing work.) With the estimated, unconnected non-ERV positions, the ILP is executed and the optimal ERV path and connected non-ERV final positions are recorded. Next, the ILP is re-executed with the actual, unconnected non-ERV data (instead of the estimated, unconnected non-ERV data) after imposing the previously obtained ERV path and the connected non-ERV relaxed final positions (through the addition of new constraints). Instead of assigning the connected non-ERV exactly at their instructed final positions generated using the ILP with the estimated, unconnected non-ERVs' positions, each connected non-ERV is allowed to adjust its final position by one extra cell in the longitudinal direction when it is evident that the ERV path is obstructed by vehicles. This test evaluates whether it is feasible for the actual, unconnected non-ERVs to safely stop at a cell that is not utilized by any connected non-ERV without jeopardiz-
ing the no passing/no weaving rule and without affecting the ERV optimal path and while following the behavior of their leader. It is an indirect assessment of whether the proposed model is acting as an efficient and optimal reservation approach with partial market penetration. Figure 3 shows the percentage of connected non-ERVs that had to adjust their longitudinal positions by one cell to accommodate the actual, unconnected vehicles without impacting the ERV intra-link path and speed, while ensuring that the actual, unconnected vehicles are following the behavior of their leader and not weaving nor passing others. At $\mathrm{v} / \mathrm{c}$ ratios of 0.75 and 0.85 , the percentage of connected vehicles that have to adjust their assigned position decreases with the increase in market penetration level. This is expected as the presence of unconnected vehicles should have less impact on the connected vehicles' behavior when the proportion of connected-to-unconnected vehicles increases. However, at high v/c ratios and at $90 \%$ market penetration, connected vehicles are closer to each other; so, when one connected non-ERV adjusts its position by one cell to accommodate an actual, unconnected non-ERV, many other connected non-ERVs are forced to do the same, which can be described as a domino effect. Therefore, at a v/c ratio of 0.95 , a higher percentage of vehicles have to adjust their positions when the market penetration level is at $90 \%$ than when it is at $80 \%$. On the other hand, as observed in Figure 3, the percentage of connected non-ERVs that adjust their instructed final positions increases when shifting from a v/c ratio of 0.75 to 0.85 for all market penetration levels. The number of unconnected vehicles along the link increases with a higher v/c ratio; so more estimation errors are expected, hence leading to a higher percentage of connected non-ERVs that needed to adjust their instructed final positions. A similar trend is observed when shifting from a v/c ratio of 0.85 to 0.95 for a $70 \%$ market penetration, also due to the error linked to the unconnected vehicles' estimation. At higher v/c ratios and market penetration levels ( $80 \%$ and $90 \%$ ), fewer positions that can fit unconnected vehicles are present, leading to less estimation error. This error reduction results in a slight decrease in the percentage of connected vehicles adjusting their final positions while shifting from a v/c ratio of 0.85 to 0.95 at a market penetration of $80 \%$. However, this trend is not observed when moving from a v/c ratio of 0.85 to 0.95 at a market penetration level of $90 \%$, as it is masked by the previously discussed domino effect.

## Test D: Comparison to current practice

In this section, the ERV intra-link path generated by the proposed system is compared to a currently adopted practice where downstream vehicles go to the nearest edge upon detecting an approaching ERV. In this case, downstream non-ERVs do not always act cooperatively and each non-ERV seeks an empty cell on its closest edge after its corresponding minimum stopping distance. The ERV's optimal intra-link path (which is the same for all tested levels of market penetration) is compared to the one that can be completed by the ERV under the current practice. The results, in Table 2, show the considerable ERV travel time reductions in seconds increasing as the v/c ratio increases. This is along a 1827ft length of combined ARs that corresponds to the $1575-\mathrm{ft}$ length of combined IRs. Note that the ERV travel times are computed based on the optimal ERV speed variables that are maximized in the objective function. These speed variables are integers and are expressed in speed stages and not in units of distance per time, to ensure comfortable acceleration and deceleration rates. After determining the actual ERV speed in distance per time at each increment using the lookup table available in the Appendix in (Hannoun et al., 3), the average


FIGURE 3 : Percentage of connected non-ERVs that adjusted their instructed final longitudinal position for different $\mathrm{v} / \mathrm{c}$ ratios and market penetration levels

ERV travel times per increment are computed and then summed to find the overall ERV travel time along the link. Under the current practice, the ERV entering the link segment on the rightmost lane is forced to make a left maneuver to avoid the non-ERVs stopped at the right edge of the road, hence decreasing its speed. In addition, the ERV continues its movement that is inevitably adjacent to stopped non-ERVs on the edges, inhibiting it from increasing its speed. Besides, as non-ERVs are not receiving any assistance and only trying to reach the nearest edge as soon as possible, passing and weaving among vehicles is expected. A risky interaction between two non-ERVs is considered when the two non-ERV are passing each other as they are heading to the same edge. For instance, four, two and eight risky interactions are noted for the scenarios with v/c ratios of $0.75,0.85$ and 0.95 respectively. On the contrary, the proposed approach avoids all types of vehicular interaction, which is by itself an added value. When traditional warning systems exist (sirens and lights), downstream non-ERVs receive limited time to react and interactions between the ERV and non-ERVs can occur, negatively impacting the ERV movement and speeds. For comparison purposes, it is assumed that the presence of an advanced emergency warning system (under the current practice) that notifies the downstream non-ERVs of an approaching ERV early in time so that the non-ERVs are stopped at the edges at the time the ERV arrives, limiting the interaction between the ERV and moving downstream non-ERVs.

## CONCLUSIONS

In this paper, an approach that optimizes the ERV movement on a link was proposed, extending the previous work of Hannoun et al. (3). Their approach, defining the optimal intra-link path that can be travelled at maximum speed and with maximum adjacent space from non-ERVs after instructing the downstream traffic to stop at specific positions was

TABLE 2: ERV travel time reduction due to the proposed system

| $\mathbf{v} / \mathbf{c}$ | ERV travel time improvement (in seconds) |
| :---: | :---: |
| 0.75 | 5.04 |
| 0.85 | 8.33 |
| 0.95 | 20.47 |

impractical on a large link due to large ILP problem size and computation time. Such challenges are overcome in this work by adopting a sequential approach to optimize the ERV movement on larger link segments. The sequential optimization technique consists of applying the ILP on shorter link segments with fewer non-ERVs consecutively, controlling the time overhead of each ILP optimization. In addition, partial market penetration is considered. The system estimates the presence of unconnected vehicles and uses that information in the optimization to reserve space for those present on the link. The estimation technique does not assume the presence of sensors and only uses the connected non-ERVs' positions. When generating the optimal final position of each non-ERV using the ILP, it is assumed that each estimated, unconnected non-ERV follows the behavior of its respective leader. The space that is virtually reserved for the estimated, unconnected non-ERVs is available for the actual, unconnected non-ERVs' use.

Future works involve the extension of this approach to make it applicable on a network-wide basis. Heavy vehicles with different sizes will be included. Also, the proposed model will be compared to practices that reflect different behaviors of non-ERVs (e.g., more cooperative actions when moving to the nearest edge). In addition, multiple emergencies in the network and/or multiple ERVs on the same link at the same point in time and contraflow use will be considered. Assessing the implications of imperfect measurements and communications and lower market penetration levels of connected vehicle technologies are part of future works, as a step forward towards deployment.

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